

Chapter 5 Notes **(Expressions)**

5.1 – Algebraic Expressions

5.3 – Properties of Operations

5.4 – The Distributive Property

5.5 – Simplifying Algebraic Expressions

5.6 – Add Linear Expressions

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Lesson 5.1 (Algebraic Expressions)

- **Variable** – a symbol that represents an unknown quantity
- **Algebraic Expression** – an expression that contains variables, numbers, and at least one operation. (There is *NO* equal sign!)

1. Give an example of an algebraic expression.

$$1 + 2$$

2. The expression $(F - 32) * \frac{5}{9}$ can be used to convert a temperature from Fahrenheit to Celsius. In this algebraic expression, which variable represents the temperature in degrees Fahrenheit?

F

- The branch of mathematics that involves expressions with variables is algebra.
 - In algebra, the multiplication sign is often omitted.
 - Different ways to represent multiplication:
6d 9st mn 6(2) 9.5.2 (m)(n)
- **Coefficient** – the numerical factor of a multiplication expression that contains a variable
 - *Hint: This is the number in front of the letter*

3. What is the coefficient in the following expression: 6d

6

4. Evaluate $2(n + 3)$ if $n = -4$.

$$\begin{aligned} 2(-4 + 3) \\ 2(-1) = \boxed{-2} \end{aligned}$$

5. Evaluate $8w - 2v$ if $w = 5$ and $v = 3$.

$$\begin{aligned} 8(5) - 2(3) \\ 40 - 6 = \boxed{34} \end{aligned}$$

6. Evaluate $4y^3 + 2$ if $y = 3$.

$$\begin{aligned} 4(3)^3 + 2 &= 4(27) + 2 \\ &= \boxed{110} \end{aligned}$$

7. Athletic trainers use the formula $\frac{3(220-a)}{5}$, where a is a person's age, to find minimum training heart rate. Find Lori's minimum training heart rate if she is 15 years old.

$$\frac{3(220-15)}{5} = \frac{3(205)}{5} = \frac{615}{5} = \boxed{123} \text{ bpm}$$

Writing Expressions:

- To translate a verbal phrase into an algebraic expression, the first step is to define a variable.
- **Defining a Variable** – choosing a variable to represent an unknown quantity

8. Marisa wants to buy a DVD player that costs \$150. She has already saved \$25 and plans to save an additional \$10 each week.

- a. Write an expression that represents the total amount of money Marisa has saved after any number of weeks. $w = \# \text{ of weeks}$

$$\boxed{25 + 10w}$$

- b. Will Marisa have saved enough money to buy the DVD player in 11 weeks?

$$25 + 10(11) = \$135$$

No, she will still need \$15 more.

9. An MP3 player costs \$70 and song downloads cost \$0.85 each.

- a. Write an expression that represents the cost of the MP3 player and x number of downloaded songs. $x = \# \text{ of songs}$

$$\boxed{70 + 0.85x}$$

- b. Find the total cost if 20 songs are downloaded.

$$70 + 0.85(20)$$

$$= \boxed{\$87}$$

Lesson 5.3 (Properties of Operations)

- **Property** – a statement that is true for any number
- **Counterexample** – an example that shows a statement or conjecture is *false*

The **Commutative Property** states that the order in which numbers are added or multiplied does not change the sum or product.

Addition

$a + b = b + a$

$6 + 1 = 1 + 6$

Multiplication

$a \cdot b = b \cdot a$

$7 \cdot 3 = 3 \cdot 7$

The **Associative Property** states that the way in which numbers are grouped when they are added or multiplied does not change the sum or product.

Addition

$a + (b + c) = (a + b) + c$

$2 + (3 + 8) = (2 + 3) + 8$

Multiplication

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

$3 \cdot (4 \cdot 5) = (3 \cdot 4) \cdot 5$

Property	Words	Symbols	Examples
Additive Identity	When 0 is added to any number, the sum is the number.	$a + 0 = a$ $0 + a = a$	$9 + 0 = 9$ $0 + 9 = 9$
Multiplicative Identity	When any number is multiplied by 1, the product is the number.	$a \cdot 1 = a$ $1 \cdot a = a$	$5 \cdot 1 = 5$ $1 \cdot 5 = 5$
Multiplicative Property of Zero	When any number is multiplied by 0, the product is 0.	$a \cdot 0 = 0$ $0 \cdot a = 0$	$8 \cdot 0 = 0$ $0 \cdot 8 = 0$

1. Name the property shown by each statement.

a. $2 * (5 * n) = (2 * 5) * n$

Associative Property (Multiplication)

b. $42 + x + y = 42 + y + x$

Commutative Property (Addition)

c. $3x + 0 = 3x$

Additive Identity

2. State whether each conjecture is true or false. If false, provide a counterexample.

a. Division of whole numbers is commutative.

False

$\frac{10}{2} = 5$

$\frac{2}{10} = \frac{1}{5}$

b. The difference of two different whole numbers is always less than both of the two numbers.

True

3. Alana wants to buy a sweater that costs \$38, sunglasses that cost \$14, a pair of jeans that costs \$22, and a T-shirt that costs \$16. Use mental math to find the total cost before tax.

$$\begin{aligned}
 &38 + 14 + 22 + 16 \\
 &(38 + 22) + (14 + 16) \\
 &60 + 30 = \boxed{\$90}
 \end{aligned}$$

4. Lance made four phone calls from his cell phone today. The calls lasted 4.7, 9.4, 2.3, and 10.6 minutes. Use mental math to find the total amount of time he spent on the phone.

$$\begin{aligned}
 &4.7 + 9.4 + 2.3 + 10.6 \\
 &(4.7 + 2.3) + (9.4 + 10.6) \\
 &7 + 20 = \boxed{27}
 \end{aligned}$$

5. Simplify each expression. Justify each step.

$$\begin{aligned}
 \text{a. } (7 + g) + 5 &= (g + 7) + 5 && \text{Commutative} \\
 &= g + (7 + 5) && \text{Associative} \\
 &= \boxed{g + 12} && \text{Simplify}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } (m * 11) * m &= (11 * m) * m && \text{Commutative} \\
 &= 11 * (m * m) && \text{Associative} \\
 &= \boxed{11m^2} && \text{Simplify}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } 4 * (3c * 2) &= 4 * (2 * 3c) && \text{Commutative} \\
 &= (4 * 2) * 3c && \text{Associative} \\
 &= 8 * 3c && \text{Simplify} \\
 &= (8 * 3) * c && \text{Associative} \\
 &= \boxed{24c} && \text{Simplify}
 \end{aligned}$$

Lesson 5.4 (The Distributive Property)

- **Equivalent Expressions** – expressions that have the same value no matter what x is
 - Example: $2(x + 2)$ and $2x + 4$

The **Distributive Property** states that to multiply a sum or difference by a number, multiply each term inside the parentheses by the number outside the parentheses.

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

$$4(6 + 2) = 4 \cdot 6 + 4 \cdot 2 \qquad 3(7 - 5) = 3 \cdot 7 - 3 \cdot 5$$

1. Use the distributive property to evaluate: $8(-9 + 4)$

$$8(-9) + 8(4)$$

$$-72 + 32 = \boxed{-40}$$

2. Use the distributive property to rewrite each expression.

a. $4(x + 7)$ $\boxed{4x + 28}$

f. $6(a + 4)$ $\boxed{6a + 24}$

b. $6(p - 5)$ $\boxed{6p - 30}$

g. $8(m + 3n)$ $\boxed{8m + 24n}$

c. $-2(x - 8)$ $\boxed{-2x + 16}$

h. $-3(y - 10)$ $\boxed{-3y + 30}$

d. $5(-3x + 7y)$ $\boxed{-15x + 35y}$

i. $\frac{1}{2}(w - 4)$ $\boxed{\frac{1}{2}w - 2}$

e. $\frac{1}{3}(x - 6)$ $\boxed{\frac{1}{3}x - 2}$

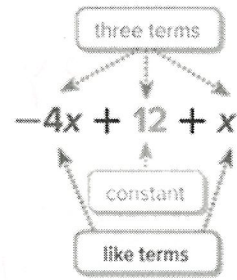
3. A sports club rents dirt bikes for \$37.50 each. Find the total cost for the club to rent 20 bikes. Justify your answer by using the distributive property.

$$20(37.50) = 20(37 + .50)$$

$$= 740 + 10 = \boxed{\$750}$$

Lesson 5.5 (Simplifying Algebraic Expressions)

- **Terms** – parts of an algebraic expression that are separated by Addition or Subtraction
- **Coefficient** – the number in front of a term that contains a variable
- **Like Terms** – terms that contain the **same variables** with the **same powers**
 - Example: $3x^2$ and $-7x^2$
 - Example: $8xy^2$ and $12xy^2$
 - Are $10x^2z$ and $22xz^2$ like terms?



- **Constant** – a term without a variable (a regular number)
 - Constant terms are also *like terms*
- **Simplest Form** – an algebraic expression with **no like terms** and **no parentheses**

Like Terms have the same variables with the same powers!!

1. Identify the terms, like terms, coefficients, and constants in each expression:

a. $6n - 7n - 4 + n$.

Terms: $6n, -7n, -4, n$

Like Terms: $6n, -7n, n$

Coefficients: $6, -7, 1$

Constants: -4

b. $9y - 4 - 11y + 7$

Terms: $9y, -4, -11y, 7$

Like Terms: $(9y, -11y) + (-4, 7)$

Coefficients: $9, -11$

Constants: $-4, 7$

c. $3x + 2 - 10 - 3x$

Terms: $3x, 2, -10, -3x$

Like Terms: $(3x, -3x); (2, -10)$

Coefficients: $3, -3$

Constants: $2, -10$

2. Write each expression in **simplest form**.

a. $4y + y$

$$5y$$

d. $6 - 3n + 3n$

$$6$$

b. $7x - 2 - 7x + 6$

$$4$$

e. $2g - 3 + 11 - 8g$

$$-6g + 8$$

c. $4z - z$

$$3z$$

3. The cost of a jacket j after a 5% markup can be represented by the expression: $j + 0.05j$

a. Simplify the expression.

$$1.05j$$

b. Determine the total cost of the jacket after the markup, if the original price is \$35.

$$\cancel{35} + 1.05(35) = \boxed{\$36.75}$$

c. Write an expression in simplest form for the cost of the jacket if the markup is 8%.

$$j + .08j = \boxed{1.08j}$$

Lesson 5.6 (Add Linear Expressions)

- **Linear Expression** – an algebraic expression in which the variable is raised to the first power

<u>Linear Expressions</u>	<u>Nonlinear Expressions</u>
$5x$	$5x^2$
$3x + 2$	

1. Add the linear expressions in each example.

a. $(2x + 3) + (x + 4)$

$$3x + 7$$

e. $(2x - 3) + (-x + 4)$

$$x + 1$$

b. $(2x - 1) + (x - 5)$

$$3x - 6$$

f. $2(x + 3) + (3x + 1)$

$$2x + 6 + (3x + 1)$$

$$5x + 7$$

c. $(3x - 5) + (2x - 3)$

$$5x - 8$$

g. $5(x - 4) + (2x - 7)$

$$5x - 20 + (2x - 7)$$

$$7x - 27$$

d. $(2x - 4) + (3x - 7)$

$$5x - 11$$

h. $6(x + 7) + (x + 3)$

$$6x + 42 + (x + 3)$$

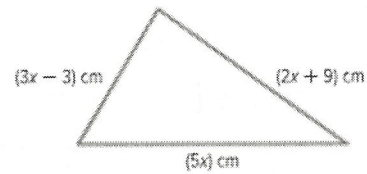
$$7x + 45$$

2. Use the triangle with the given side lengths.

- a. Write a linear expression in **simplest form** to represent the perimeter of the triangle.

$$3x - 3 + 5x + 2x + 9$$

$$\boxed{10x + 6}$$



- b. Find the perimeter if the value of x is 5 centimeters.

$$10(5) + 6 = \boxed{56 \text{ cm}}$$

3. A rectangle has side lengths $(x + 4)$ feet and $(2x - 2)$ feet.

- a. Write a linear expression in **simplest form** to represent the perimeter.

$$2(x + 4) + 2(2x - 2)$$

$$2x + 8 + 4x - 4$$

$$\boxed{6x + 4}$$

- b. Find the perimeter if the value of x is 7 feet.

$$6(7) + 4 = \boxed{46 \text{ ft}}$$

Lesson 5.7 (Subtract Linear Expressions)

- When subtracting linear expressions, subtract like terms.
 - Method: Add the opposite (the additive inverse).

1. Subtract the linear expressions in each example.

a. $(6x + 3) - (2x + 2)$

$$4x + 1$$

e. $(6x - 10) - (2x - 8)$

$$4x - 2$$

b. $(2x - 3) - (x - 2)$

$$x - 1$$

f. $(-2x - 4) - (2x)$

$$-4x - 4$$

c. $(-2x - 4) - (2x)$

$$-4x - 4$$

g. $(3x - 2) - (5x - 4)$

$$-2x + 2$$

d. $(5x - 9) - (2x - 7)$

$$3x - 2$$

h. $(4x - 4) - (-2x + 2)$

$$6x - 6$$

i. $(6x + 5) - (3x + 1)$

$$3x + 4$$

k. $(4x - 3) - (2x + 7)$

$$2x - 10$$

j. $(-4x - 7) - (-5x - 2)$

$$x - 5$$

l. $(5x - 4) - (2x + 3)$

$$3x - 7$$

2. A hat store tracks the sale of college and professional team hats for m months. The number of college hats sold is represented by $(6m + 3)$. The number of professional hats sold is represented by $(5m - 2)$.

- a. Write an expression to show how many more college hats were sold than professional hats.

$$(6m + 3) - (5m - 2)$$

$$\boxed{m + 5}$$

- b. Evaluate the expression if m equals 10.

$$10 + 5 = \boxed{15}$$