

Chapter 3.1 Notes (Graphing Linear Equations)

Objectives:

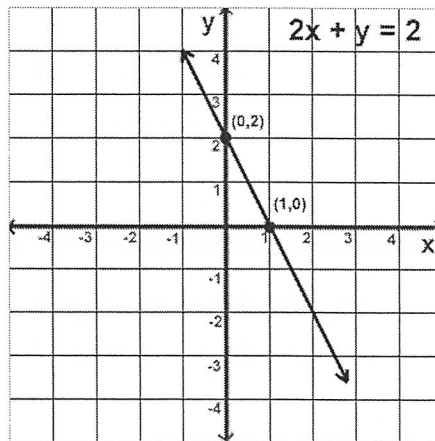
- Identify linear equations, intercepts, and zeros.
- Graph linear equations.

Linear Equation – forms a *line* when graphed

- No exponents on variables (other than 1)
- No variables in denominator
- No xy terms

Standard Form: $Ax + By = C$

- $A \geq 0$
- $A, B,$ and C are integers (no fractions or decimals)
- A and B are *not both zero*



- **Examples:** Determine whether each equation is linear. Then, convert to standard form (if possible).

1. $y = 4 - 3x$ *yes*

$$\begin{array}{r} y = 4 - 3x \\ +3x \quad +3x \\ \hline 3x + y = 4 \end{array}$$

3. $\frac{1}{3}y = -1$ *yes*

$$3 \cdot \frac{1}{3}y = -1 \cdot 3$$

$$y = -3$$

2. $6x - xy = 4$

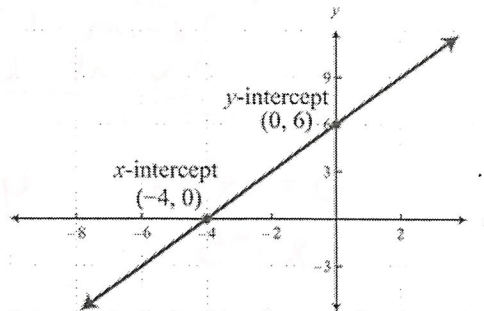
no

4. $y = x^2 - 4$

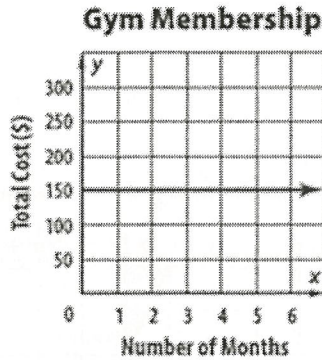
no

Intercepts

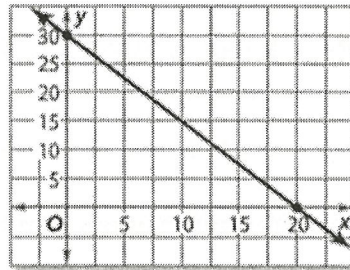
- **X - Intercept:** point at which graph crosses the x-axis (when $y=0$)
- **Y - Intercept:** point at which graph crosses the y-axis (when $x=0$)



- Examples: Find the x- and y-intercepts for each graph.



x: none
y: 150
(0, 150)



x: 20
(20, 0)
y: 30
(0, 30)

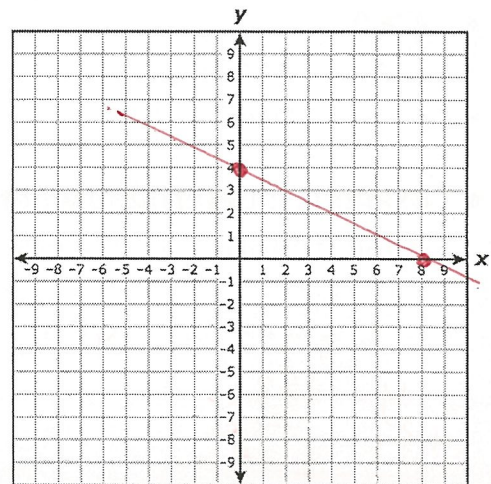
Graph Using Intercepts – only 2 points are needed to graph a line

- To find the x-intercept, let $y=0$.
- To find the y-intercept, let $x=0$.
- Example: Graph $2x + 4y = 16$ by using the x- and y-intercepts.

x: $2x + 4(0) = 16$
 $2x = 16$
 $x = 8$

y: $2(0) + 4y = 16$
 $4y = 16$
 $y = 4$

x-int: (8, 0)
y-int: (0, 4)



- Practice: Find the x- and y-intercepts for each equation. Then, graph each equation.

1. $-x + 2y = 3$

$-(0) + 2y = 3$
 $2y = 3$
 $y = 3/2$

$-x + 2(0) = 3$
 $-x = 3$
 $x = -3$

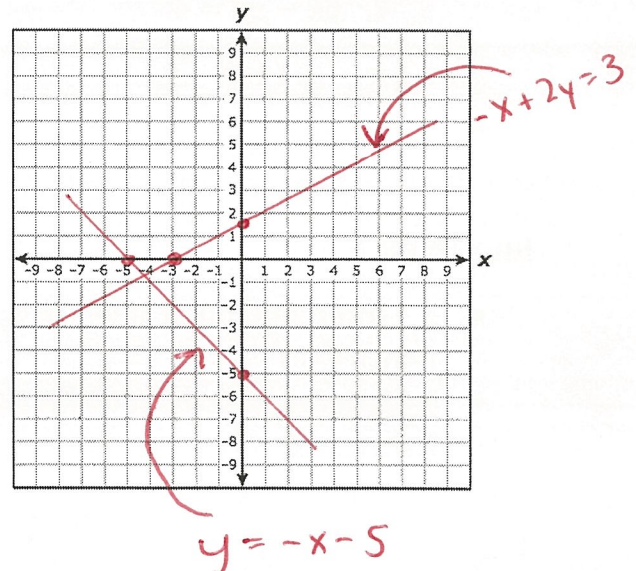
x-int: (-3, 0)
y-int: (0, 3/2)

2. $y = -x - 5$

$0 = -x - 5$
 $x = -5$

$y = -(0) - 5$
 $y = -5$

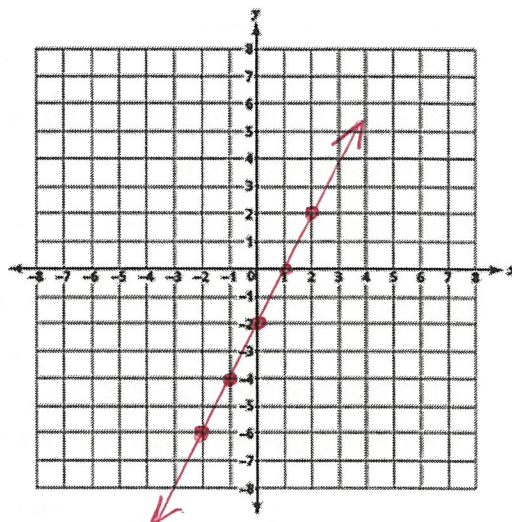
x-int: (-5, 0)
y-int: (0, -5)



Graph Using a Table

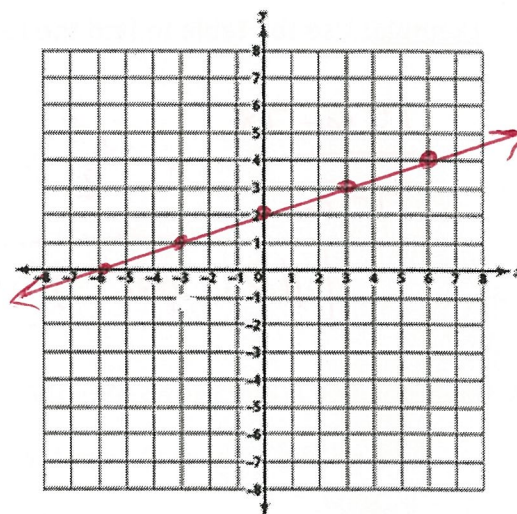
1. $2x - y = 2 \Rightarrow y = 2x - 2$

x	$y = 2x - 2$	y	(x, y)
-2	$2(-2) - 2$	-6	(-2, -6)
-1	$2(-1) - 2$	-4	(-1, -4)
0	$2(0) - 2$	-2	(0, -2)
1	$2(1) - 2$	0	(1, 0)
2	$2(2) - 2$	2	(2, 2)



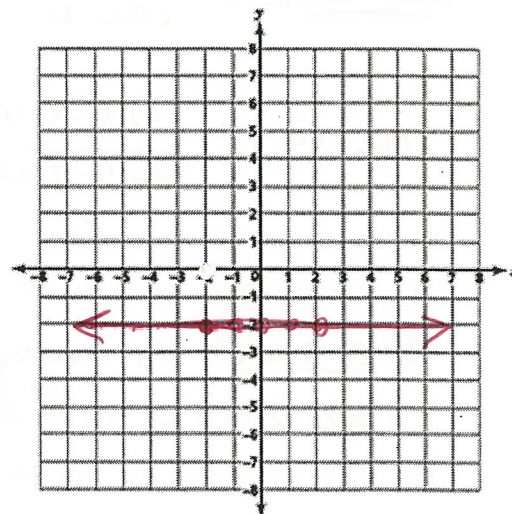
2. $y = \frac{1}{3}x + 2$

x	$y = \frac{1}{3}x + 2$	y	(x, y)
-6	$\frac{1}{3}(-6) + 2$	0	(-6, 0)
-3	$\frac{1}{3}(-3) + 2$	1	(-3, 1)
0	$\frac{1}{3}(0) + 2$	2	(0, 2)
3	$\frac{1}{3}(3) + 2$	3	(3, 3)
6	$\frac{1}{3}(6) + 2$	4	(6, 4)



3. $y = -2$

x	$y = -2$	y	(x, y)
-2	-2	-2	(-2, -2)
-1	-2	-2	(-1, -2)
0	-2	-2	(0, -2)
1	-2	-2	(1, -2)
2	-2	-2	(2, -2)



Chapter 3.3 Notes (Rate of Change and Slope)

Objectives:

- Use rate of change to solve problems.
- Find the slope of a line.

Rate of Change – ratio that describes how one quantity changes with respect to another

KeyConcept Rate of Change

If x is the independent variable and y is the dependent variable, then

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

- **Example:** Use the table to find the rate of change. Then, explain its meaning.

$$\frac{156 - 78}{4 - 2} = \frac{78}{2} = 39$$

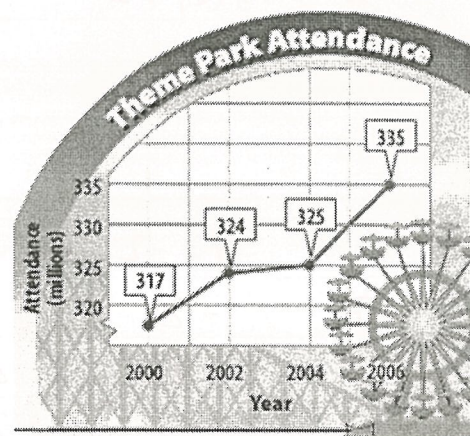
\$39 per game

Number of Computer Games	Total Cost (\$)
x	y
2	78
4	156
6	234

- **Practice:** The graph shows the number of people who visited U.S. theme parks in recent years.
 - Find the rates of change for 2000-2002 and 2002-2004.

$$\underline{2000-2002}: \frac{324 - 317}{2002 - 2000} = \frac{7}{2} = 3.5$$

$$\underline{2002-2004}: \frac{325 - 324}{2004 - 2002} = \frac{1}{2}$$



- Explain the meaning of each rate of change.

2000-2002: On average, 3.5 million more people per year

2002-2004: On average, 0.5 million more people per year

- Without calculating, find the 2-year period that has the least rate of change.

2002 - 2004

****Remember: All linear functions have a constant rate of change!!**

- The change in x-values must be constant.
- The change in y-values must be constant.

• Examples: Determine whether each function is linear. Explain.

a)

x	y
1	-6
4	-8
7	-10
10	-12

$\downarrow -2$
 $\downarrow -2$
 $\downarrow -2$

Linear

b)

x	y
-2	4
-1	1
0	-1
1	-4

$\downarrow -3$
 $\downarrow -2$
 $\downarrow -3$

NOT Linear

c)

x	y
0	2
1	4
2	8
3	16

$\downarrow +2$
 $\downarrow +4$
 $\downarrow +8$

NOT Linear

Slope – number that describes the *steepness* and *direction* of a line

- Slope is usually denoted by m .
- Slope can be found using “rise over run.”

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

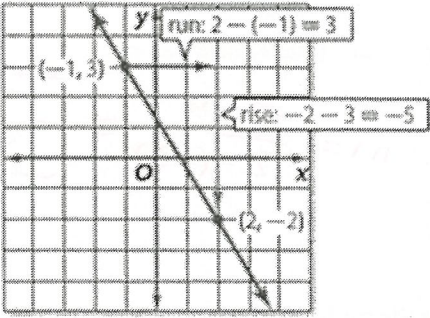
The graph shows a line that passes through $(-1, 3)$ and $(2, -2)$.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

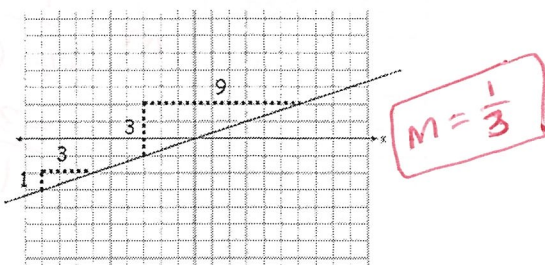
$$= \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$= \frac{-2 - 3}{2 - (-1)} \text{ or } -\frac{5}{3}$$

So, the slope of the line is $-\frac{5}{3}$.



[Example 1] Find Slope Using a Graph

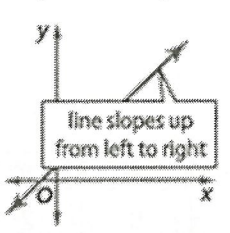
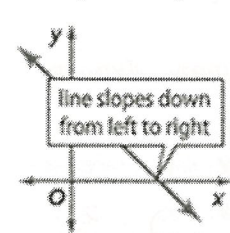
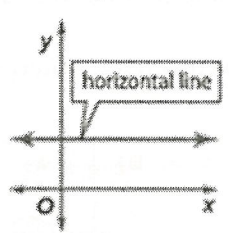
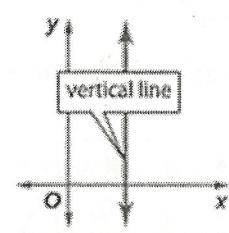


[Example 2] Find Slope Using the Slope Formula

Find the slope between the points $(-9, -3)$ and $(6, 2)$.

$$m = \frac{2 - (-3)}{6 - (-9)} = \frac{5}{15} = \frac{1}{3}$$

4 Types of Slope:

<p>positive slope</p>  <p>The function values are increasing over the entire domain.</p>	<p>negative slope</p>  <p>The function values are decreasing over the entire domain.</p>	<p>slope of 0</p>  <p>The function values are constant over the entire domain.</p>	<p>undefined slope</p>  <p>The relation is not a function.</p>
--	--	---	--

“HOY – VUX”

HOY – Horizontal Lines have Zero Slope and cross the y-axis

VUX – Vertical Lines have Undefined Slope and cross the x-axis

- Examples: Find the slope of the line that passes through each pair of points.

1. (-2, 0) and (1, 5)

$$m = \frac{5-0}{1-(-2)} = \boxed{\frac{5}{3}}$$

3. (-3, -1) and (2, -1)

$$m = \frac{-1-(-1)}{2-(-3)} = \frac{0}{5} = \boxed{0}$$

2. (-3, 4) and (2, -3)

$$m = \frac{-3-4}{2-(-3)} = \boxed{-\frac{7}{5}}$$

4. (-2, 4) and (-2, -3)

$$m = \frac{-3-4}{-2-(-2)} = \frac{-7}{0} = \boxed{\text{Undefined}}$$

Finding Slope in Tables:

- Examples: Determine the slope of each linear equation. (Hint: Find the rate of change.)

a)

x	y
0	5
1	7
2	9
3	11

$$m = \frac{7-5}{1-0} = \boxed{2}$$

b)

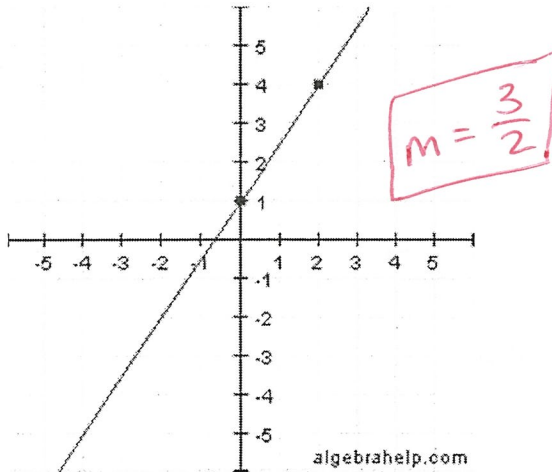
x	y
-7	10
-6	7
-5	4
-4	1

$$m = \frac{7-10}{-6-(-7)} = \frac{-3}{1} = \boxed{-3}$$

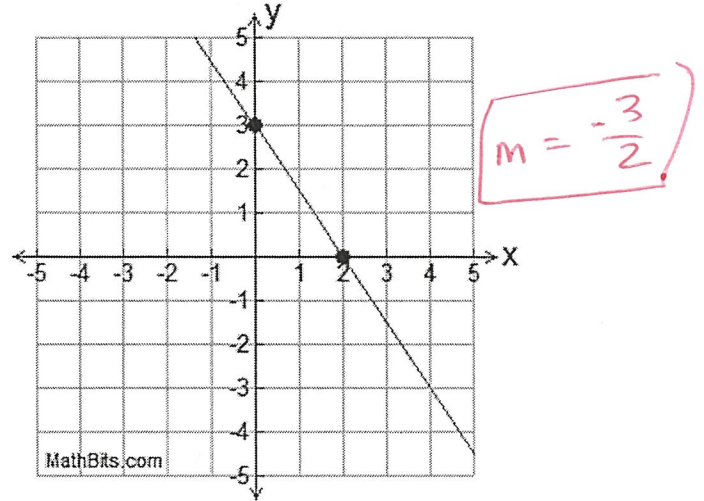
Finding Slope in Graphs:

- Examples: Determine the slope of each linear equation. (Do not use the slope formula.)

a)



b)

**Application of the Slope Formula: Find Coordinates Given the Slope**

- Example: Find the value of r so that the line through $(1, 4)$ and $(-5, r)$ has a slope of $\frac{1}{3}$.

$$\frac{1}{3} = \frac{r-4}{-5-1} \Rightarrow \frac{1}{3} = \frac{r-4}{-6}$$

$$-6(1) = 3(r-4)$$

$$-6 = 3r - 12$$

$$6 = 3r$$

$$\boxed{2 = r}$$

- Practice: Find the value of r so the line that passes through each pair of points has the given slope.

1. $(-2, 6)$ and $(r, -4)$; $m = -5$

$$-5 = \frac{-4-6}{r-(-2)}$$

$$-5 = \frac{-10}{r+2}$$

$$-5(r+2) = -10$$

$$-5r - 10 = -10$$

$$-5r = 0$$

$$\boxed{r = 0}$$

2. $(r, 6)$ and $(4, 8)$; $m = 2$

$$2 = \frac{8-6}{4-r}$$

$$2 = \frac{2}{4-r}$$

$$2(4-r) = 2$$

$$8 - 2r = 2$$

$$-2r = -6 \Rightarrow \boxed{r = 3}$$

