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Lesson 8.5 Notes (Using the Distributive Property)

Use the Distributive Property to Factor – You can work backward to express a polynomial as the product of a monomial factor and a polynomial factor.

Factoring by Grouping – terms are put into groups and then factored (used to factor polynomials with *4 or more terms*)

Multiplying	Factoring
$3(a + b) = 3a + 3b$	$3a + 3b = 3(a + b)$
$x(y - z) = xy - xz$	$xy - xz = x(y - z)$
$6y(2x + 1) = 6y(2x) + 6y(1)$ $= 12xy + 6y$	$12xy + 6y = 6y(2x) + 6y(1)$ $= 6y(2x + 1)$

Example 1: Factor $12mp + 80m^2$.

Find the GCF of $12mp$ and $80m^2$.

Write each term as the product of the GCF and its remaining factors.

$12mp + 80m^2 = 4m(3p) + 4m(20m)$

$= \mathbf{4m(3p + 20m)}$

Example 2: Factor $6ax + 3ay + 2bx + by$ by grouping.

$6ax + 3ay + 2bx + by$

$= (6ax + 3ay) + (2bx + by)$

$= 3a(2x + y) + b(2x + y)$

$= \mathbf{(3a + b)(2x + y)}$

Exercises:

Factor each polynomial.

1. $24x + 48y$

$24(x+2y)$

2. $30mp^2 + m^2p - 6p$

$p(30mp + m^2 - 6)$

3. $q^4 - 18q^3 + 22q$

$q(q^3 - 18q^2 + 22)$

4. $14t^3 - 42t^5 - 49t^4$

$7t^3(2 - 6t^2 - 7t)$

5. $55p^2 - 11p^4 + 44p^5$

$11p^2(5 - p^2 + 4p^3)$

6. $14y^3 - 28y^2 + y$

$y(14y^2 - 28y + 1)$

7. $x^2 + 2x + x + 2$ $x(x+1) + 2(x+1)$

$\boxed{(x+1)(x+2)}$

8. $6y^2 - 4y + 3y - 2$ $3y(2y+1) - 2(2y+1)$

$(2y+1)(3y-2)$

9. $4m^2 + 4mp + 3mp + 3p^2$ $4m(m+p) + 3p(m+p)$

$(4m+3p)(m+p)$

10. $12ax + 3xz + 4ay + yz$ $4a(3x+y) + z(3x+y)$

$\boxed{(3x+y)(4a+z)}$

11. $12a^2 + 3a - 8a - 2$ $3a(4a+1) - 2(4a+1)$

$(4a+1)(3a-2)$

12. $xa + ya + x + y$ $a(x+y) + 1(x+y)$

$(x+y)(a+1)$

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Solve Equations by Factoring The Zero Product Property, along with factoring, can be used to solve equations.

Zero Product Property	For any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0, *b* = 0, or both *a* and *b* equal 0.

Example: Solve $9x^2 + x = 0$. Then check the solutions.

Write the equation so that it is of the form $ab = 0$.

$9x^2 + x = 0$	Original equation
$x(9x + 1) = 0$	Factor the GCF of $9x^2 + x$, which is x.
$x = 0$ or $9x + 1 = 0$	Zero Product Property
$x = 0 \qquad x = -\frac{1}{9}$	Solve each equation.

The solution set is $\left\{0, -\frac{1}{9}\right\}$.

Exercises:

Solve each equation. Check your solutions.

13. $x(x + 3) = 0$ (0, -3)

14. $3m(m - 4) = 0$ (0, 4)

15. $(r - 3)(r + 2) = 0$ (-2, 3)

16. $3x(2x - 1) = 0$ (0, ½)

17. $(4m + 8)(m - 3) = 0$ {-2, 3}

18. $5t^2 = 25t$ {0, 5}
$5t^2 - 25t = 0$
$5t(t - 5) = 0$

19. $12x^2 = -6x$ $\{-\frac{1}{2}, 0\}$
$12x^2 + 6x = 0$
$6x(2x + 1) = 0$

20. $(4a + 3)(8a + 7) = 0$ $\{-\frac{7}{8}, -\frac{3}{4}\}$

21. $8y = 12y^2$ $\{0, \frac{2}{3}\}$
$12y^2 - 8y = 0$
$4y(3y - 2) = 0$

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Lesson 8.6 Notes (Solving $x^2 + bx + c = 0$)

Factor $x^2 + bx + c$

- Find two integers, m and p, whose sum is equal to b and whose product is equal to c.
- When c is positive, its factors have the same signs.
  - If b is positive, the factors are positive. If b is negative, the factors are negative.
- When c is negative, its factors have opposite signs.
  - The factor with the greater absolute value has the same sign as b.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + p)$, where $m + p = b$ and $mp = c$

Example 1: Factor $x^2 + 7x + 10$.

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $p = 5$.

$x^2 + 7x + 10 = \mathbf{(x + 5)(x + 2)}$

Example 2: Factor $x^2 + 6x - 16$.

In this trinomial, $b = 6$ and $c = -16$. This means $m + p$ is positive and mp is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of –16	Sum of Factors
1, –16	–15
–1, 16	15
2, –8	–6
–2, 8	6

Therefore, $m = -2$ and $p = 8$.

$x^2 + 6x - 16 = \mathbf{(x - 2)(x + 8)}$

Exercises:

Factor each polynomial.

1. $x^2 + 4x + 3$

$(x+3)(x+1)$

2. $m^2 + 12m + 32$

$(m+4)(m+8)$

3. $r^2 - 3r + 2$

$(r-2)(r-1)$

4. $x^2 - x - 6$

$(x-3)(x+2)$

5. $x^2 - 4x - 21$

$(x-7)(x+3)$

6. $x^2 - 22x + 121$

$(x-11)(x-11)$

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Factor each polynomial.

7. $t^2 - 4t - 12$

$(t+2)(t-6)$

8. $p^2 - 16p + 64$

$(p-8)(p-8)$

9. $9 - 10x + x^2$ $x^2 - 10x + 9$

$\cancel{(9-x)(1-x)}$

$(x-9)(x-1)$

10. $x^2 + 6x + 5$

$(x+5)(x+1)$

11. $a^2 + 8a - 9$

$(a-1)(a+9)$

12. $y^2 - 7y - 8$

$(y-8)(y+1)$

<u>Quadratic Equation</u> – can be written in the standard form $ax^2 + bx + c = 0$

- Factoring and the Zero Product Property can be used to solve many equations of the form $x^2 + bx + c = 0$.

<u>Example</u>: Solve $x^2 + 6x = 7$. Check your solutions.

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x-1)(x+7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1 \quad x = -7$	Solve each equation.

The solution set is **{1, –7}**.

<u>Exercises:</u>

Solve each equation. Check the solutions.

13. $x^2 - 4x + 3 = 0$ $\{1, 3\}$

$(x-3)(x-1)=0$

14. $y^2 - 5y + 4 = 0$ $\{1, 4\}$

$(y-4)(y-1)=0$

15. $m^2 + 10m + 9 = 0$ $\{-1, -9\}$

$(m+9)(m+1)=0$

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Solve each equation. Check the solutions.

16. $x^2 = x + 2$ $\{-1, 2\}$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$

17. $x^2 - 4x = 5$ $\{-1, 5\}$
$x^2 - 4x - 5 = 0$
$(x-5)(x+1) = 0$

18. $x^2 - 12x + 36 = 0$ $\{6\}$
$(x-6)(x-6) = 0$

19. $t^2 - 8 = -7t$ $\{-8, 1\}$
$t^2 + 7t - 8 = 0$
$(t+8)(t-1) = 0$

20. $p^2 = 9p - 14$ $\{2, 7\}$
$p^2 - 9p + 14 = 0$
$(p-7)(p-2) = 0$

21. $-9 - 8x + x^2 = 0$ $\{-1, 9\}$
$x^2 - 8x - 9 = 0$
$(x-9)(x+1) = 0$

Real-world Applications:

The formula $h = vt - 16t^2$ gives the height h of a rocket after t seconds when the initial velocity v is given in feet per second. Use the formula $h = vt - 16t^2$ to solve each problem.

22. A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to first reach a height of 32 feet? 1 second

23. If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high? 10 sec
90 sec

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Lesson 8.7 Notes (Solving $ax^2 + bx + c = 0$)

Factor $ax^2 + bx + c$

- Find two integers m and p whose product is equal to ac and whose sum is equal to b.
- If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1: Factor $2x^2 + 15x + 18$.

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers that have a sum of 15 and a product of $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + px + c$, with $a = 2$, $m = 3$, $p = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= \mathbf{(x + 6)(2x + 3)} \end{aligned}$$

Example 2: Factor $3x^2 - 3x - 18$.

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF. $3x^2 - 3x - 18 = 3(x^2 - x - 6)$. Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of –6 with a sum of –1.

Factors of –6	Sum of Factors
1, –6	–5
–1, 6	5
–2, 3	1
2, –3	–1

Now use the pattern $(x + m)(x + p)$ with $m = 2$ and $p = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is
$3x^2 - 3x - 18 = \mathbf{3(x + 2)(x - 3)}$

Exercises:

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$

$(2x+1)(x-2)$

2. $3m^2 - 8m - 3$

$(3m+1)(m-3)$

3. $16r^2 - 8r + 1$

$(4r-1)(4r-1)$

4. $6x^2 + 5x - 6$

$(2x+3)(3x-2)$

5. $3x^2 + 2x - 8$

$(3x-4)(x+2)$

6. $18x^2 - 27x - 5$

$(3x-5)(6x+1)$

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7. $3y^2 - 6y - 24$

$3(y^2-2y-8)$

$\boxed{3(y+2)(y-4)}$

8. $4x^2 + 26x - 48$

$2(2x^2+13x-24)$

$\boxed{2(x+8)(2x-3)}$

9. $8m^2 - 44m + 48$

$4(2m^2-11m+12)$

$\boxed{4(2m-3)(m-4)}$

10. $6x^2 - 7x + 18$

$\boxed{\text{prime}}$

Product : 108

Sum : −7

11. $2a^2 - 14a + 18$

$2(a^2-7a+9)$

12. $18 + 11y + 2y^2$

$\boxed{\text{prime}}$

Product : 36

Sum : 11

Solve Equations by Factoring: **Solve each equation.**

13. $8x^2 + 2x - 3 = 0$ $\{\frac{1}{2}, -\frac{3}{4}\}$

14. $3n^2 - 2n - 5 = 0$ $\{-1, \frac{5}{3}\}$

15. $2d^2 - 13d - 7 = 0$ $\{-\frac{1}{2}, 7\}$

16. $2k^2 - 40 = -11k$ $\{-8, \frac{5}{2}\}$

17. $2p^2 = -21p - 40$ $\{-\frac{5}{2}, -8\}$

18. $-7 - 18x + 9x^2 = 0$ $\{\frac{7}{3}, -\frac{1}{3}\}$

Real-world Application:

19. The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions? 30yd * 10 yd

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Lesson 8.8 Notes (Differences of Squares)

Difference of Two Squares – a binomial expression in the form $a^2 - b^2$

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = \mathbf{(a + b)(a - b)}$

Example 1: Factor $50a^2 - 72$.

$50a^2 - 72$

$= 2(25a^2 - 36)$ Find the GCF.

$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$

$= \mathbf{2(5a + 6)(5a - 6)}$ Factor the difference of squares.

Example 2: Factor $4x^4 + 8x^3 - 4x^2 - 8x$.

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial

$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.

$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.

$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.

$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.

$= \mathbf{4x(x - 1)(x + 1)(x + 2)}$ Factor the difference of squares.

Exercises:

Factor each polynomial.

1. $x^2 - 81$

$(x+9)(x-9)$

2. $m^2 - 100$

$(m+10)(m-10)$

3. $16n^2 - 25$

$(4n-5)(4n+5)$

4. $36x^2 - 100y^2$

$4(9x^2-25y^2)$

$4(3x+5y)(3x-5y)$

5. $49x^2 - 36$

$(7x+6)(7x-6)$

6. $16a^2 - 9b^2$

$(4a-3b)(4a+3b)$

7. $225b^2 - a^2$

$(15b-a)(15b+a)$

8. $72p^2 - 50$

$2(36p^2-25)$

$\boxed{2(6p+5)(6p-5)}$

9. $-2 + 2x^2$ $= 2x^2-2$

$2(x^2-1)$

$\boxed{2(x+1)(x-1)}$

10. $-81 + a^4$

a^4-81

$(a^2+9)(a^2-9)$

$\boxed{(a^2+9)(a+3)(a-3)}$

11. $6 - 54a^2$ $= -54a^2+6$

$-6(9a^2-1)$

$\boxed{-6(3a+1)(3a-1)}$

12. $8y^2 - 200$

$8(y^2-25)$

$\boxed{8(y+5)(y-5)}$

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13. $169x^3 - x$

$x(169x^2-1)$

$\boxed{x(13x+1)(13x-1)}$

14. $3a^4 - 3a^2$

$3a^2(a^2-1)$

$\boxed{3a^2(a+1)(a-1)}$

15. $3x^4 + 6x^3 - 3x^2 - 6x$

$3x(x^3+2x^2-x-2)$

$3x(x^2(x+2)-1(x+2))$

$3x(x^2-1)(x+2)$

$\boxed{3x(x+1)(x-1)(x+2)}$

<u>Solve Equations by Factoring</u>: Solve $x^2 - \frac{1}{25} = 0$.

$x^2 - \frac{1}{25} = 0$	Original equation
$x^2 - \left(\frac{1}{5}\right)^2 = 0$	$x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$	Factor the difference of squares.
$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$	Zero Product Property
$x = -\frac{1}{5}$ or $x = \frac{1}{5}$	Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$.

<u>Exercises:</u>

Solve each equation by factoring. Check the solutions.

16. $81x^2 = 49$

$81x^2-49=0$

$(9x+7)(9x-7)=0$

$\boxed{\left\{-\frac{7}{9}, \frac{7}{9}\right\}}$

17. $36n^2 = 1$

$36n^2-1=0$

$(6n+1)(6n-1)=0$

$\boxed{\left\{-\frac{1}{6}, \frac{1}{6}\right\}}$

18. $25d^2 - 100 = 0$

$25(d^2-4)=0$

$25(d+2)(d-2)=0$

$\boxed{\{-2, 2\}}$

19. $\frac{1}{4}x^2 = 25$ $\{10, -10\}$

20. $36 = \frac{1}{25}x^2$ $\{-30, 30\}$

21. $\frac{49}{100} - x^2 = 0$ $\left\{-\frac{7}{10}, \frac{7}{10}\right\}$

22. $16y^3 = 25y$ $\left\{0, -\frac{5}{4}, \frac{5}{4}\right\}$

23. $\frac{1}{64}x^2 = 49$ $\{-56, 56\}$

24. $4a^3 - 64a = 0$ $\{0, -4, 4\}$

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Lesson 8.9 Notes (Perfect Squares)

Perfect Square Trinomial – a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$

The trinomial $16x^2 + 24x + 9$ is a perfect square trinomial, as illustrated below.

$$16x^2 + 24x + 9$$

Is the first term a perfect square? Yes, because $16x^2 = (4x)^2$

Is the middle term twice the product of the square roots of the first and last terms? Yes, because $24x = 2(4x)(3)$.

Is the last term a perfect square? Yes, because $9 = 3^2$

KeyConcept Factoring Perfect Square Trinomials

Symbols	$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
	$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$
Examples	$x^2 + 8x + 16 = (x + 4)(x + 4)$ or $(x + 4)^2$
	$x^2 - 6x + 9 = (x - 3)(x - 3)$ or $(x - 3)^2$

Example 1: Determine whether $16n^2 - 24n + 9$ is a perfect square trinomial. If so, factor it.

Since $16n^2 = (4n)(4n)$, it is a perfect square.

Since $9 = 3 \cdot 3$, the last term is a perfect square.

The middle term is equal to $2(4n)(3)$.

$16n^2 - 24n + 9$ is a perfect square trinomial.

$16n^2 - 24n + 9 = (4n)^2 - 2(4n)(3) + 3^2$

$= \mathbf{(4n - 3)^2}$

Exercises:

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $x^2 - 16x + 64$ **2.** $m^2 + 10m + 25$ **3.** $p^2 + 8p + 64$

Selecting the Correct Factoring Method:

ConceptSummary Factoring Methods

Steps	Number of Terms
Step 1 Factor out the GCF.	any
Step 2 Check for a difference of squares or a perfect square trinomial.	2 or 3
Step 3 Apply the factoring patterns for $x^2 + bx + c$ or $ax^2 + bx + c$ (general trinomials), or factor by grouping.	3 or 4

Example 2: Factor $16x^2 - 32x + 15$.

Since 15 is not a perfect square, use a different factoring pattern.

$16x^2 - 32x + 15$	Original trinomial
$= 16x^2 + mx + px + 15$	Write the pattern.
$= 16x^2 - 12x - 20x + 15$	$m = -12$ and $p = -20$
$= (16x^2 - 12x) - (20x - 15)$	Group terms.
$= 4x(4x - 3) - 5(4x - 3)$	Find the GCF.
$= \mathbf{(4x - 5)(4x - 3)}$	Factor by grouping.