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Lesson 8.1 Notes (Adding and Subtracting Polynomials)

- **Polynomial** – a monomial or sum of monomials
  - o **Binomial** – sum of 2 monomials
  - o **Trinomial** – sum of 3 monomials

Monomial	Binomial	Trinomial
$5x$	$2x^2 + 7$	$x^3 - 10x + 1$

- **Degree of a monomial** – the sum of the exponents of all its variables
- **Degree of a polynomial** – the greatest degree of any term in the polynomial

- **Standard form of a polynomial** – terms are arranged so that the terms are in order from greatest *degree to* least *degree*
  - o **Leading Coefficient** – the coefficient of the first term

leading coefficient greatest degree
Standard form: $4x^3 - 5x^2 + 2x + 7$

Example: Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*. Then find the degree of the polynomial.

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	**Yes**. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	**Yes**. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	**No**. $3n^{-4} = \frac{3}{n^4}$, which is not a monomial	---	---
$9x^3 + 4x + x + 4 + 2x$	**Yes**. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises:

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

1. 36 Yes ; 0 ; monomial

2. $\frac{3}{q^2} + 5$ No

3. $7x - x + 5$ Yes ; 1 ; binomial
$6x + 5$

4. $8g^2h - 7gh + 2$ Yes ; 3 ; trinomial

5. $\frac{1}{4y^2} + 5y - 8$ No

6. $6x + x^2$ Yes ; 2 ; binomial

Write each polynomial in standard form. Identify the leading coefficient.

7. $x^3 + x^5 - x^2$ $x^5 + x^3 - x^2$ 1

8. $x^4 + 4x^3 - 7x^5 + 1$ $-7x^5 + x^4 + 4x^3 + 1$ -7

9. $2x^7 - x^8$ $-x^8 + 2x^7$ -1

10. $3x + 5x^4 - 2 - x^2$ $5x^4 - x^2 + 3x - 2$ 5

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Add and Subtract Polynomials

- To add polynomials, you can group like terms horizontally or align like terms vertically.
- **Like terms** – monomial terms that have the same variables with corresponding variables having the same exponents (*Examples*: $3p$ and $-5p$ or $2x^2y$ and $8x^2y$)
- You can subtract a polynomial by adding its additive inverse. (*Hint*: Replace each term with its opposite.)

Example: Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$
$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$
$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$
$= 2x^2 + (-9)$
$= 2x^2 - 9$

The difference is $\mathbf{2x^2 - 9}$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 3x^2 + 2x - 6 \\ \underline{(-)} & \underline{x^2 + 2x + 3} \\ & 3x^2 + 2x - 6 \\ \underline{(+)} & \underline{-x^2 - 2x - 3} \\ & 2x^2 \qquad - 9 \end{array}$$

The difference is $\mathbf{2x^2 - 9}$.

Exercises

Find each sum or difference.

11. $(6xy + 2y + 6x) + (4xy - x)$

$10xy + 2y + 5x$

12. $(x^2 + y^2) + (-x^2 + y^2)$

$2y^2$

13. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$

$4p^2 - 9p + 10$

14. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$

$-4x^2 + 6y^2 + 4xy$

15. $(3x^2 - 2x) - (3x^2 + 5x - 1)$

$(3x^2 - 2x) + (-3x^2 - 5x + 1)$

$\boxed{-7x + 1}$

16. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$

$(4x^2 + 6xy + 2y^2) + (x^2 - 2xy + 5y^2)$

$\boxed{5x^2 + 4xy + 7y^2}$

17. $(2h - 6j - 2k) - (-7h - 5j - 4k)$

$(2h - 6j - 2k) + (7h + 5j + 4k)$

$\boxed{9h - j + 2k}$

18. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$

$(9xy^2 + 5xy) + (2xy + 8xy^2)$

$\boxed{17xy^2 + 7xy}$

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Lesson 8.2 Notes (Multiplying a Polynomial by a Monomial)

Polynomial Multiplied by Monomial

- The Distributive Property can be used to multiply a polynomial by a monomial.
  - Sometimes multiplying results in like terms.
  - The products can be simplified by combining like terms.

Example 1: Find $-3x^2(4x^2 + 6x - 8)$.

$-3x^2(4x^2 + 6x - 8)$

$= -3x^2(4x^2) + (-3x^2)(6x) - (-3x^2)(8)$

$= -12x^4 + (-18x^3) - (-24x^2)$

$= \mathbf{-12x^4 - 18x^3 + 24x^2}$

Example 2: Simplify $-2(4x^2 + 5x) - x(x^2 + 6x)$.

$-2(4x^2 + 5x) - x(x^2 + 6x)$

$= -2(4x^2) + (-2)(5x) + (-x)(x^2) + (-x)(6x)$

$= -8x^2 + (-10x) + (-x^3) + (-6x^2)$

$= (-x^3) + [-8x^2 + (-6x^2)] + (-10x)$

$= \mathbf{-x^3 - 14x^2 - 10x}$

Exercises

Find each product.

1. $x(5x + x^2)$

$5x^2 + x^3$

2. $x(4x^2 + 3x + 2)$

$4x^3 + 3x^2 + 2x$

3. $-2xy(2y + 4x^2)$

$-4xy^2 - 8x^3y$

4. $-4ax(10 + 3x)$

$-40ax - 12ax^2$

5. $3y(-4x - 6x^3 - 2y)$

$-12xy - 18x^3y - 6y^2$

6. $2x^2y^2(3xy + 2y + 5x)$

$6x^3y^3 + 4x^2y^3 + 10x^3y^2$

Simplify each expression.

7. $6a(2a - b) + 2a(-4a + 5b)$

$4a^2 + 4ab$

8. $4r(2r^2 - 3r + 5) + 6r(4r^2 + 2r + 8)$

$32r^3 + 68r$

9. $-2z(4z^2 - 3z + 1) - z(3z^2 + 2z - 1)$

$-11z^3 + 4z^2 - z$

10. $2(4x^2 - 2x) - 3(-6x^2 + 4) + 2x(x - 1)$

$28x^2 - 6x - 12$

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Equations with Polynomial Expressions

- Many equations contain polynomials that must be added, subtracted, or multiplied before the equation can be solved.

Example: Solve $4(n - 2) + 5n = 6(3 - n) + 19$.

$4(n - 2) + 5n = 6(3 - n) + 19$	Original equation
$4n - 8 + 5n = 18 - 6n + 19$	Distributive Property
$9n - 8 = 37 - 6n$	Combine like terms.
$15n - 8 = 37$	Add $6n$ to each side.
$15n = 45$	Add 8 to each side.
$n = 3$	Divide each side by 15.

Exercises

Solve each equation.

11. $3x(x - 5) - 3x^2 = -30$ 2

12. $6(x^2 + 2x) = 2(3x^2 + 12)$ 2

13. $4(2p + 1) - 12p = 2(8p + 12)$ -1

14. $2(6x + 4) + 2 = 4(x - 4)$ -3.25

15. $-2(4y - 3) - 8y + 6 = 4(y - 2)$ 1

16. $x(x + 2) - x(x - 6) = 10x - 12$ 6

17. $3(2a - 6) - (-3a - 1) = 4a - 2$ 3

18. $5(2x^2 - 1) - (10x^2 - 6) = -(x + 2)$ -3

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Lesson 8.3 Notes (Multiplying Polynomials)

Multiply Binomials

- To multiply two binomials, you can apply the Distributive Property twice.
  - A useful way to keep track of terms in the product is to use the FOIL method.

To multiply two binomials, find the sum of the products of *F* the *First* terms, *O* the *Outer* terms, *I* the *Inner* terms, *L* and the *Last* terms.

		Product of First Terms		Product of Outer Terms		Product of Inner Terms		Product of Last Terms
$(x+4)(x-2)$	=	$(x)(x)$	+	$(x)(-2)$	+	$(4)(x)$	+	$(4)(-2)$
	=	$x^2 - 2x + 4x - 8$						
	=	$x^2 + 2x - 8$						

Example: Find $(x-2)(x+5)$.

$(x-2)(x+5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$

$= x^2 + 5x + (-2x) - 10$

$= x^2 + 3x - 10$

The product is $\mathbf{x^2 + 3x - 10}$.

Exercises

Find each product.

1. $(p-4)(p+2)$

$p^2 - 2p - 8$

2. $(y+5)(y+2)$

$y^2 + 7y + 10$

3. $(2x-1)(x+5)$

$2x^2 + 9x - 5$

4. $(3x+1)(4x+3)$

$12x^2 + 13x + 3$

5. $(x-8)(-3x+1)$

$-3x^2 + 25x - 8$

6. $(5t+4)(2t-6)$

$\cancel{10t^2 + 2}$

$10t^2 - 22t - 24$

7. $(5m-3n)(4m-2n)$

$\cancel{20m^2 - 22}$

$20m^2 - 10mn - 12mn + 6n^2$

$\boxed{20m^2 - 22mn + 6n^2}$

8. $(a-3b)(2a-5b)$

$2a^2 - 5ab - 6ab + 15b^2$

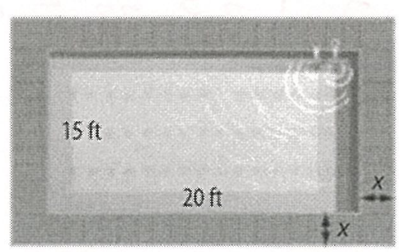
$\boxed{2a^2 - 11ab + 15b^2}$

9. $(8x-5)(8x+5)$

$64x^2 + 40x - 40x - 25$

$\boxed{64x^2 - 25}$

10. A contractor is building a deck around a rectangular pool. The deck is x feet from each side of the pool. Write an expression for the total area of the pool and deck.



$A = l \cdot w$

$= (2x+20)(2x+15)$

$= 4x^2 + 30x + 40x + 300$

$= \boxed{4x^2 + 70x + 300}$

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Multiply Polynomials

- The Distributive Property can be used to multiply any two polynomials.

Example: Find (3x + 2)(2x² – 4x + 5).

$(3x + 2)(2x^2 - 4x + 5)$

$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$	Distributive Property
$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$	Distributive Property
$= 6x^3 - 8x^2 + 7x + 10$	Combine like terms.

The product is $\mathbf{6x^3 - 8x^2 + 7x + 10}$.

Exercises

Find each product.

11. $(3n - 4)(n^2 + 5n - 4)$

$3n^3 + 11n^2 - 32n + 16$

12. $(8x - 2)(3x^2 + 2x - 1)$

$24x^3 + 10x^2 - 12x + 2$

13. $(2a + 4)(2a^2 - 8a + 3)$

$4a^3 - 8a^2 - 26a + 12$

14. $(3x - 4)(2x^2 + 3x + 3)$

$6x^3 + x^2 - 3x - 12$

15. $(n^2 + 2n - 1)(n^2 + n + 2)$

$n^4 + 3n^3 + 3n^2 + 3n - 2$

16. $(t^2 + 4t - 1)(2t^2 - t - 3)$

$2t^4 + 7t^3 - 9t^2 - 11t + 3$

17. $(y^2 - 5y + 3)(2y^2 + 7y - 4)$

$2y^4 - 3y^3 - 33y^2 + 41y - 12$

18. $(3b^2 - 2b + 1)(2b^2 - 3b - 4)$

$6b^4 - 13b^3 - 4b^2 + 5b - 4$

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Lesson 8.4 Notes (Special Products)

Squares of Sums and Differences

- Some pairs of binomials have products that follow specific patterns.

Square of a Sum	$(a + b)^2 = (a + b)(a + b) = \mathbf{a^2 + 2ab + b^2}$
Square of a Difference	$(a - b)^2 = (a - b)(a - b) = \mathbf{a^2 - 2ab + b^2}$

Example 1: Find $(3a + 4)(3a + 4)$.

Use the square of a sum pattern, with $a = 3a$ and $b = 4$.

$(3a + 4)(3a + 4) = (3a)^2 + 2(3a)(4) + (4)^2$

$= 9a2 + 24a + 16$

The product is $\mathbf{9a^2 + 24a + 16}$.

Example 2: Find $(2z - 9)(2z - 9)$.

Use the square of a difference pattern with $a = 2z$ and $b = 9$.

$(2z - 9)(2z - 9) = (2z)^2 - 2(2z)(9) + (9)(9)$

$= 4z^2 - 36z + 81$

The product is $\mathbf{4z^2 - 36z + 81}$.

Exercises

Find each product.

1. $(x - 6)^2$

$x^2 - 12x + 36$

2. $(3p + 4)^2$

$9p^2 + 24p + 16$

3. $(4x - 5)^2$

$16x^2 - 40x + 25$

4. $(2x - 1)^2$

$4x^2 - 4x + 1$

5. $(2h + 3)^2$

$4h^2 + 12h + 9$

6. $(m + 5)^2$

$m^2 + 10m + 25$

7. $(x^3 - 1)^2$

$x^6 - 2x^3 + 1$

8. $(2h^2 - k^2)^2$

$4h^4 - 4h^2k^2 + k^4$

9. $\left(\frac{1}{4}x + 3\right)^2$

$\frac{1}{16}x^2 + \frac{3}{2}x + 9$

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Example: Real-world Application

10. Alano has a garden that is g feet long and g feet wide. He wants to add 3 feet to the length and the width.

a. Show how the new area of the garden can be modeled by the square of a binomial.

$A = s^2 = \boxed{(g+3)^2}$

b. Find the square of this binomial.

$(g+3)^2 = \boxed{g^2 + 6g + 9}$

Product of a Sum and a Difference

- There is also a pattern for the product of a sum and a difference of the same two terms, $(a + b)(a - b)$.
  - The product is called the **difference of** squares.

Product of a Sum and a Difference	$(a + b)(a - b) = a^2 - ab + ab - b^2 = \mathbf{a^2 - b^2}$

Example: Find $(5x + 3y)(5x - 3y)$.

$(a + b)(a - b) = a^2 - b^2$	Product of a Sum and a Difference
$(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2$	$a = 5x$ and $b = 3y$
$= 25x^2 - 9y^2$	Simplify.

The product is $25x^2 - 9y^2$.

Exercises

Find each product.

11. $(2x - 1)(2x + 1)$

$4x^2 - 1$

12. $(h + 7)(h - 7)$

$h^2 - 49$

13. $(m - 5)(m + 5)$

$m^2 - 25$

14. $(y - 4x)(y + 4x)$

$y^2 - 16x^2$

15. $(8 + 4x)(8 - 4x)$

$64 - 16x^2$

16. $(3a - 2b)(3a + 2b)$

$9a^2 - 4b^2$

17. $(x^3 - 2)(x^3 + 2)$

$x^6 - 4$

18. $(h^2 - k^2)(h^2 + k^2)$

$h^4 - k^4$

19. $\left(\frac{1}{4}x + 2\right)\left(\frac{1}{4}x - 2\right)$

$\frac{1}{16}x^2 - 4$