

Lesson 7.3 Notes (Rational Exponents)**Objectives:**

- Evaluate and rewrite expressions involving rational exponents.
- Solve equations involving expressions with rational exponents.

Rational Exponents:

- **Exponential Form** – number or variable raised to a power ($x^{1/n}$)
- **Radical Form** – number or variable under a radical symbol ($\sqrt[n]{x}$)

Square Root	$b^{\frac{1}{2}} = \sqrt{b}$
Cube Root	$b^{\frac{1}{3}} = \sqrt[3]{b}$
<i>n</i>th Root	$b^{\frac{1}{n}} = \sqrt[n]{b}$

Square Roots: Write each expression in radical form, or write each radical in exponential form.

1. $14^{\frac{1}{2}}$ $\sqrt{14}$

2. $5x^{\frac{1}{2}}$ $5\sqrt{x}$

3. $\sqrt{17}$ $17^{\frac{1}{2}}$

4. $\sqrt{12n}$ $(12n)^{\frac{1}{2}}$
= $2(3n)^{\frac{1}{2}}$

5. $19ab^{\frac{1}{2}}$ $19a\sqrt{b}$

6. $\sqrt{18b}$ $(18b)^{\frac{1}{2}}$
= $3(2b)^{\frac{1}{2}}$

***n*th Roots:** Simplify each expression.

7. $\sqrt[3]{27}$ (3)

8. $\sqrt[5]{32}$ (2)

9. $\sqrt[4]{10,000}$ (10)

10. $\sqrt[3]{64}$ (4)

11. $125^{\frac{1}{3}}$ (5)

12. $256^{\frac{1}{4}}$ (4)

13. $1296^{\frac{1}{4}}$ (6)

Extension:

$$b^{\frac{m}{n}} = (\sqrt[n]{b})^m \text{ or } \sqrt[n]{b^m}$$

The Power of a Power property allows us to extend the definition of $b^{\frac{1}{n}}$ to $b^{\frac{m}{n}}$.

$$b^{\frac{m}{n}} = (b^{\frac{1}{n}})^m \quad \text{Power of a Power}$$

$$= (\sqrt[n]{b})^m \text{ or } \sqrt[n]{b^m} \quad b^{\frac{1}{n}} = \sqrt[n]{b}$$

Examples: Simplify each expression.

$$14. 64^{\frac{2}{3}} \quad (64^{\frac{1}{3}})^2$$

$$= (4)^2 = \textcircled{16}$$

$$15. 36^{\frac{3}{2}} \quad (36^{\frac{1}{2}})^3$$


$$= (6)^3 = \textcircled{216}$$

$$16. 27^{\frac{2}{3}} \quad (27^{\frac{1}{3}})^2$$

$$= (3)^2 = \textcircled{9}$$

Exponential Equation – an equation in which variables occur as exponents

- Use the Power Property of Equality and the other properties of exponents to solve exponential equations.

 **Key Concept Power Property of Equality**

Words For any real number $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

Examples If $5^x = 5^3$, then $x = 3$. If $n = \frac{1}{2}$, then $4^n = 4^{\frac{1}{2}}$.

Examples: Solve each equation.

$$17. 2^x = 128$$

$$2^x = 2^7$$

$$x = \textcircled{7}$$

$$18. 3^{3x+1} = 81$$

$$3^{3x+1} = 3^4$$

$$3x+1 = 4$$

$$3x = 3$$

$$x = \textcircled{1}$$

$$19. 5^x = 125$$

$$5^x = 5^3$$

$$x = \textcircled{3}$$

$$20. 6^{3x+2} = 216$$

$$6^{3x+2} = 6^3$$

$$3x+2 = 3$$

$$3x = 1$$

$$x = \textcircled{\frac{1}{3}}$$