

Lesson 6.2 Notes (Substitution)

Objectives:

- Solve systems of equations by using substitution.
- Solve real-world problems involving systems of equations by using substitution.

• **Substitution Method:**

KeyConcept Solving by Substitution

Step 1 When necessary, solve at least one equation for one variable.

Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Example: Use substitution to solve each system of equations.

1. $\begin{cases} y = 2x + 1 \\ 3x + y = -9 \end{cases}$

$3x + 2x + 1 = -9$
 $5x + 1 = -9$
 $5x = -10$
 $x = -2$

$y = 2x + 1$
 $y = 2(-2) + 1$
 $y = -4 + 1$
 $y = -3$

$(-2, -3)$

2. $\begin{cases} x + 2y = 6 \\ 3x - 4y = 28 \end{cases} \Rightarrow x = 6 - 2y$

$3(6 - 2y) - 4y = 28$
 $18 - 6y - 4y = 28$
 $18 - 10y = 28$
 $-10y = 10$
 $y = -1$

$x = 6 - 2y$
 $x = 6 - 2(-1)$
 $x = 6 + 2$
 $x = 8$

$(8, -1)$

Practice: Use substitution to solve each system of equations.

3. $\begin{cases} 4x + 5y = 11 \\ y - 3x = -13 \end{cases} \Rightarrow y = -13 + 3x$

$4x + 5(-13 + 3x) = 11$
 $4x - 65 + 15x = 11$
 $19x - 65 = 11$
 $19x = 76$
 $x = 4$

$y = -13 + 3x$
 $y = -13 + 3(4)$
 $y = -13 + 12$
 $y = -1$

$(4, -1)$

4. $\begin{cases} 2x + 5y = -1 \\ y = 3x + 10 \end{cases}$

$2x + 5(3x + 10) = -1$
 $2x + 15x + 50 = -1$
 $17x + 50 = -1$
 $17x = -51$
 $x = -3$

$y = 3x + 10$
 $y = 3(-3) + 10$
 $y = -9 + 10$
 $y = 1$

$(-3, 1)$

- Special Cases:**

- **No Solution** – result is a *false* statement
- **Infinite Solutions** – result is always *true* (identity)

Examples: Use substitution to solve each system of equations.

$$5. \begin{cases} y = 2x - 4 \\ -6x + 3y = -12 \end{cases}$$

$$\begin{aligned} -6x + 3y &= -12 \\ -6x + 3(2x - 4) &= -12 \\ -6x + 6x - 12 &= -12 \\ 0 - 12 &= -12 \\ -12 &= -12 \end{aligned}$$

Infinite Solutions

$$6. \begin{cases} 2x - y = 8 \\ y = 2x - 3 \end{cases}$$

$$\begin{aligned} 2x - y &= 8 \\ 2x - (2x - 3) &= 8 \\ 2x - 2x + 3 &= 8 \\ 0 + 3 &= 8 \\ 3 &= 8 \quad (\text{False}) \end{aligned}$$

No Solution

- Real-world Applications**

7. A store sold a total of 125 televisions and speakers. The televisions sold for \$104.95, and the speakers sold for \$18.95. The sales from these two items totaled \$6926.75. How many of each item was sold. $t = \# \text{ televisions}$ $s = \# \text{ speakers}$

$$\begin{cases} t + s = 125 \Rightarrow t = 125 - s \\ 104.95t + 18.95s = 6926.75 \end{cases}$$

$$\begin{aligned} 104.95(125 - s) + 18.95s &= 6926.75 \\ 13118.75 - 104.95s + 18.95s &= 6926.75 \\ 13118.75 - 86s &= 6926.75 \end{aligned}$$

$$\begin{aligned} -86s &= -6192 \\ s &= \boxed{72} \text{ speakers} \\ t &= 125 - s \\ t &= 125 - 72 = \boxed{53} \text{ televisions} \end{aligned}$$

8. As of 2009, the New York Yankees and the Cincinnati Reds together had won a total of 32 World Series. The Yankees had won 5.4 times as many as the Reds. How many World Series had each team won? $y = \text{Yankees}$ $r = \text{Reds}$

$$\begin{cases} y + r = 32 \\ y = 5.4r \end{cases}$$

$$y + r = 32$$

$$5.4r + r = 32$$

$$6.4r = 32$$

$$r = \boxed{5}$$

$$\begin{aligned} y &= 5.4r \\ y &= 5.4(5) \\ y &= \boxed{27} \end{aligned}$$

Reds: 5 games
Yankees: 27 games